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TREBALL FI DE GRAU

Grau en Enginyeria Electrònica Industrial i Automàtica

DESIGN OF A YAW RATE CONTROL FOR A FORMULA STUDENT ELECTRIC CAR

Memòria i Annex

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Resum

En aquest projecte es descriu el procés d'estudi i disseny teòric del control de velocitat en *yaw* aplicat a un vehicle elèctric amb tracció posterior independent de Formula Student. La base de disseny és el cotxe desenvolupat per l'equip de Formula Student e-Tech Racing de la EEBE durant temporada 2016-17.

Durant els primers capítols es fa una introducció que descriu quin és l'entorn de Formula Student sobre el que es treballarà, seguidament les característiques tècniques del vehicle i la base matemàtica de la dinàmica vehicular que s'utilitzarà posteriorment.

La part central del projecte detallada en els capítols 4 i 5, es presenta l'estructura de l'algoritme i el disseny del control en si mateix. En el capítol 4 s'especifica la funcionalitat de cada bloc i es mostra la estabilitat del sistema juntament amb el controlador PI proposat. En el capítol 5, s'han realitzat simulacions per diferents escenaris utilitzant senyals d'excitació com entrada esglaó, entrada en rampa i situacions més properes a la realitat com la trajectòria de un SkidPad, d'aquesta manera comprovar la sortida de parell diferencial.

Finalment, es comenten el resultats obtinguts de l'algoritme de manera crítica donant importància als seus punts forts i els seus punts *febles*.

Resumen

Este proyecto describe el estudio y diseño teórico de un control de velocidad en *yaw* para un vehículo eléctrico de tracción independiente posterior de Formula Student. La base del diseño es el coche construido completamente por el equipo de Formula Student de la EEBE e-Tech Racing durante la temporada 2016-17.

En los primeros capítulos se hace una introducción sobre que es la Formula Student y su entorno, seguidamente se presentan las principales características técnicas del vehículo y la base matemática sobre dinámica vehicular que se utiliza.

La parte central del proyecto se detalla en los capítulos 4 y 5, donde se presenta la estructura del algoritmo en sí y en diseño del control. En el capítulo 4 se especifica la funcionalidad de cada bloque y se muestra la estabilidad del sistema junto con el control PI propuesto. En el capítulo 5, se han realizado simulaciones para diferentes escenarios utilizado señales de excitación como la entrada escalón, entrada en rampa y situaciones más reales como la trayectoria de un Skid Pad, y así comprobar la salida del par diferencial.

Finalmente, los resultados obtenidos con el algoritmo se han comentado de manera crítica resaltando los puntos fuertes y las debilidades.

Abstract

This project deals with the study and design of a direct yaw rate control for a two rear independent wheel drive Formula Student electric vehicle. It takes as a base of design the 2016-17 season car fully built and developed car by the students of e-Tech Racing Formula Student team of the EEBE

First chapters are composed by an introduction of Formula Student concept and its environment, the main technical characteristics of the car and the mathematic base about vehicle dynamics

The core of the project resides in the chapters 4 and 5, where it is presented the algorithm and the controller. In chapter 4 is detailed an explanation of all different parts and is showed the system stability within the proposed PI controller. In Chapter 5, simulations of different scenarios have been done using inputs as step function, ramp o more realistic trajectory as Skip Pad.

Finally, results obtained with the controller simulations are commented critically highlighting points of strength and weakness of the solution.

Glossary

List of Acronyms

AMZ	Academic Motorsports Zurich
BMS	Battery Management System
CAN	Controlled Area Network
CG	Centre of Gravity
CS	Cornering Stiffness
DNF	Does Not Finish
EEBE	Escola d'Enginyeria de Barcelona Est
ECU	Electronic Control Unit
EUETIB	Escola Universitària d'Enginyeria Tècnica Industrial de Barcelona
FS	Formula Student
IGBT	Insulated-Gate Bipolar Transistor
LVDT	Linear Voltage Differential Transformer
LCD	Liquid Crystal Display
NS	Neutral Steer
OV	Over Steer
PCB	Printed Circuit Board
PID	Proportional-Integrative-Derivative
SAE	Society of Automotive Engineers
SOC	State of Charge
SOH	State of Health

US	Under Steer

List of Variables

Centre of Gravity i axis	CG_i	mm	always positive
Cornering Stiffness	C_i	N/rad	always negative
Driver Command	D_{comand}	adimensional	always positive
Effective Tire Radius	r_{eff}	m	always positive
Forward Velocity CG	V, u	m/s	+ for forward
Front Bias	t_f	mm	always positive
Heading Angle	β	radians	+ for slip to right
Lateral acceleration	a_y	m/s^2	+ clockwise turns
Lateral Force	F_{y_i}	N	+ to right
Lateral velocity CG	v	m/s	+ to right
Longitudinal CG location	a, b	meters	always positive
Longitudinal Force	$F_{x_{ij}}$	N	+ to forward
Mass	m_i	kg	always positive
Moment Z axis	M_z	N·m	+ clockwise
Normal Force	F_{z_i}	N	always positive
Radius of turning	R	meters	always positive
Rear Bias	t_r	mm	always positive
Slip angle	α_i	radian	+ for slip to right
Stability Factor	K	1/(m/s^2)	+ for US

Steer angle front wheels	δ	radian	+ clockwise
Torque to Wheels	T_{ij}	N·m	+ for forward
Transmission ratio	r_{trans}	adimensional	always positive
Wheelbase	l	mm	always positive
Yawing velocity	r	radians/seg	+ clockwise

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1. Introduction

1.1 Motivation

When e-Tech Racing was created in the past 2012 it was a clear differential aspect around the university in terms of self-education, very near technical application field and high technology uses. When the author of this thesis sign in (season 2014-2015) from now, it is clear how quickly the knowledge has grown in the team and the individual high level of its members.

Thanks to the feedback get in the competitions where the team participate (2014-15 & 2015-16) organised by FS Spain marshals in Montmeló circuit, perceptible improvements about dynamic behaviour of the car could be studied and applied. The aim of increase in ranking competition involve get more points for static or dynamic every events, hence the study of the project which pretends, in case of real application, be quicker than no yaw velocity control applied.

1.2 Goals

The main goal to the design is yaw control algorithm which applied in the current season car, will manage output torque of the rear wheels and power delivery. To reach it, the project has been divided in more specific objectives.

- Study of the car and dynamic model
- Design and simulation of the algorithm in different test scenarios

Real success will be accomplished with a friendly understanding, quickly implementation in software systems and notable improvement lap times when applied.

1.3 Scope and considerations

In this project can be differentiated between two clear concepts. Vehicle dynamics and control theory. A wide research has been carried out on different references that present similar systems to those that combine the previous fields. These use electric or hybrid commercial vehicle baseline, four-wheel drive vehicles or even in same FS competition. In this way, an appropriate methodology has been chosen in order the team could apply it first time ever.

As described in [7 p.125], there are different levels of model the dynamics of a vehicle. This work has focused on the most abstract level (far from reality), which at the same time is the easier to start, it keeps in mind that future members of the team will work and investigate on levels closer to reality obtaining more complex and more accurate models. The demonstrated mathematical basis is used as a feedback on the behaviour of the actual vehicle.

Based on the references discussed in the previous paragraphs, where different control methodologies are used such as sliding mode control [3], feedforward contribution with PID controller [2] or gradient method and adaptive control by reference model [1]. The work as mentioned before is intended to be a starting point to design a yaw velocity controller, therefore, the PI strategy has been chosen as one of the least risky and at the same time more robust which begin. In order to achieve good tuning, rapid and overwhelming response requirements parameters have been defined as suggested in [8] of the controller's design. Finally, it has been validated using Matlab-Simulink Software.

Considerations to keep in mind, the algorithm only will perform its functionality when higher values than 0 from accelerator pedal exists. That means, when the driver brakes or release the accelerator pedal there will be no control or negative torques.

Referring the tire, despite being aware that its behaviour is widely studied and really complex to model because of its non-linearity, in the thesis is contemplate as a linear element to generate both longitudinal and lateral forces.

2. Environment

2.1. E-Tech Racing & Formula Student

This chapters expose the history of the team, cars of actual and old seasons, and those main characteristics. A Formula Student competition briefing is also done, including what are the static and dynamic events.

2.1.1. Formula Student

Born in 80's, SAE (Society of Automotive Engineers) create an alternative model of competition with a clear mission of encourage the university students to compete and develop their own personal skills in a context of massive project management and close up the relationship between the student and the companies.

Divided in two parts, static and dynamic events, variety of international judges that work in high level technology companies, accumulate years of experience in the motorsport field allowing them to evaluate the whole project team though point scaling.

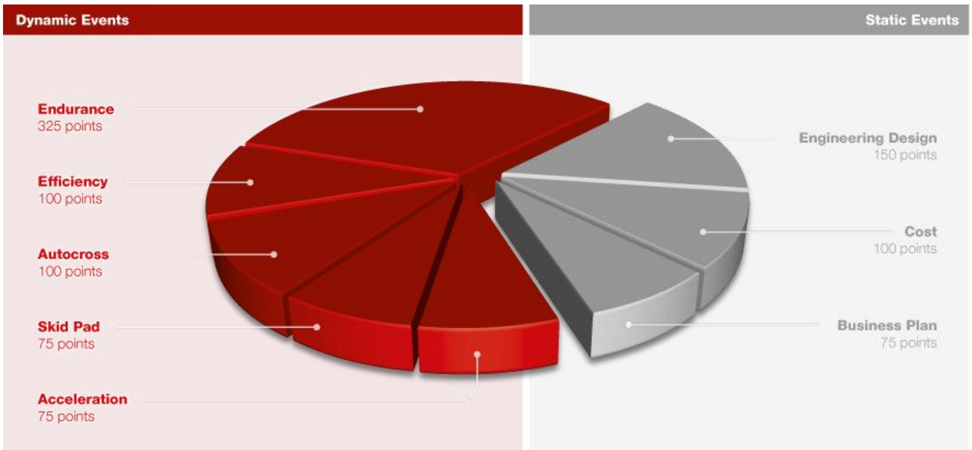

*Figure 1.Points Distribution in competition (350 pts). **Source**: AMZ Racing.*

2.1.1.1. Static Events

Static tests are performed without the car being in motion. They are the first ones that are made and ensures that the car is safe to compete.

- Design event: The judges evaluates the engineering decisions that have been taken to build the car. Divided in different departments, the team have to defend this decision and demonstrate the knowledge of the car.

- Cost report: Each team generates an Excel document with an analysis of the cost of manufacturing the whole vehicle in mass production in hypothetical case. The judges evaluate the final cost, the precision of the manufacturing cost and how successful the process used to manufacture was.

- Presentation event: A fictitious business plan is presented where the judges, who act as investors, must be persuaded to bet on a series production of the vehicle made in the current season by the team. Finally, aspects to valuate by investors are how profitable is the project, what is the target audience and the business strategy that has been followed.

- Technical Inspection or Scrutineering: in this test the "scrutineers" are in charge of checking that if the car is ready to run. On first, the scrutineers check point-by-point the vehicle complies with the current season rules, according to electric or combustion cars. Secondly, the "Tilt Test" is carried out, where the vehicle is subjected to a 45 ° and 60 ° inclination to check that it does not lose fluid and does not turn over with the driver inside. With the vehicle turned on, the electric vehicle has to pass the "Rain Test" where it is verified that the high-voltage part of the car is isolated from the rest of it and there is no risk of electrocution. For combustion category, the "Noise Test" check the exhaust cannot exceed certain decibels of noise emission. For the both disciplines, in "Brake Test" the cars must take enough speed to suddenly brake and be able to block the 4 wheels at the same time.

2.1.1.2. Dynamic Events

Once all the static events have been passed, the vehicle is certified with stickers that is allowed to perform the dynamic tests, already with the car in motion. These tests are common for both categories, but an important point to emphasize is that they are carried out individually and sequentially. There is no situation of rivalry that could affect the safety of the pilot, the car or the marshals.

- Acceleration event: This event measures the car maximum longitudinal force that is able to generate. Along a 75 metres straight line, the score is based on the time this distance is covered. The best are around 3.5 seconds.

- In Skid Pad, the maximum lateral force of the vehicle is evaluated. It takes place in an 8 shape circuit where 4 turns are made, 2 to the right and 2 to the left. The time obtained is the average of the best time right lap and best time left lap. Approximately the best times are 4.8 seconds.

- Autocross: The vehicle must travel around a circuit of 1 km marking the best lap time possible. The circuit combines sections of straight lines, open and closed curves and braking points where it is tested the dynamics of the vehicle. Points are obtained from depending this time and is used to establish the endurance start order too.

- Endurance and Efficiency: The most important test of the whole competition. It consists of 20-30 min run where 22 km are covered around the same Autocross circuit. The durability and reliability of the car is tested. In the same test, consumption is measured along this one, rewarding the car which less amount of energy spent. In the case of the electric, the kWh consumed through an "Energy Meter" assigned by the organization, and in case of combustion vehicles, the litres of fuel consumed are consulted.

*Figure 2. Stev-e in Endurance FS Czech Republic. **Source:** e-Tech Racing.*

Finally, the points of the different tests are distributed and a “Overall winner”, among others prizes, is acclaimed. This score will mark the final position in a "World Ranking" where all the Formula Student teams are in.

2.1.2. E-Tech Racing Formula Student Team

During August 2012, different EUETIB students were volunteers from the competition performed at Formula Student Spain in Montmeló circuit. The philosophy of this competition raises a great interest for these students and finally, in September 2012, a Formula Student team is born under the name of

EUETIB e-Tech Racing with the clear goal of building vehicles for the subsequent participation in the competitions as an accredited team.

In 2012/13 season the team is committed to start with electric vehicles, something very uncommon in a team that has just started, since they usually start with combustion vehicles. The season is only used to acquire knowledge and the maximum economic and logistical support. It is a season where no vehicle is built and the team does not participate in any competition

2.1.2.1. 2013/14 Season

The previous season was used to establish the bases of the team, thus they start the design and development of the first vehicle of the team. Named "E79", the car of tubular chassis and fiberglass body, was presented to the competition without power train due to the lack of financial resources suffered by the team. Only participate in the static tests, and thanks to them the team got very valuable feedback from the judges that gave them strength to improve the next season.

*Figure 3. E-79. **Source:** e-Tech Racing.*

2.1.2.2. 2014/15 Season

With a desire and a renewed team, the result of the season was an electric car called "Will-e", in honour of Albert Einstein quote that the team sponsored: "There is a driving force more power than steam, electricity and atomic energy: the will." It consisted of a 370-kilogram vehicle, with tubular steel chassis, 2-wheel drive propelled by 55-kW Mavilor MA-55 engines. In competition, the car could not pass to dynamics events.

*Figure 3. Will-e. **Source:** e-Tech Racing.*

2.1.2.3. 2015/16 Season

Focused with the objectives of weight reduction and reliability, the new design result in a car of 273 kilograms, maintaining the same power train but combining different materials to reduce weight. The team passed for the first time the static events allowing the participation in the dynamic part. However, in the absence of 4 laps to end the "Endurance", a mechanical problem left the car as DNF.

*Figure 4. eV-A FS Spain. **Source:** e-Tech Racing.*

2.1.2.4. 2016/17 Season

With notable changes to the power train, "Stev-e" incorporate tailor-made design with Vernis Motors company collaboration, of 40 kW and 130 Nm each. Inverters are swapped with Unitek Bamocar 700/400 and the package is resized to get 488.8 V and 12500 mAh. The incorporation of a complete aerodynamic package, downforce and cooling aspects are improved. The first success obtained is the first international competition that the team participates in the Czech Republic, where it is able to finish all the tests for the first time. In FS Spain the team once again repeats completing endurance confirming the reliability after a year which the team has worked so hard.

*Figure 5. Stev-e in FS Czech Republic. **Source**: e-Tech Racing*

*Figure 6. Stev-e in FS Spain. **Source:** e-Tech Racing*

2.2. The car

2.2.1. Mechanical Aspects

These are the overall dimensions of the car

Wheelbase $[l]$	1560 mm
Front track $[t_f]$	1250 mm
Rear track $[t_r]$	1150 mm
Front shaft to CG$[a]$	876 mm
Rear shaft to CG $[b]$	689 mm
Vehicle mass $[m]$	275.5 kg
Front mass $[m_f]$	120.58 kg
Rear mass $[m_r]$	154.41 kg
Front Cornering Stiffness $[C_f]$	-1000 N/rad
Rear Cornering Stiffness $[C_r]$	-1200 N/rad

Table 1. Technical data of Stev-e.

13" rims from the BRAID brand designed for Formula Student are used and Hoosier 20.5x7.0-13 R25B regard the tires. The wheels are connected with a braking system with two pistons for the front pads and a simple piston for the rear. The suspension chosen is "double Wishbone" topology with "pull-rod" for front and “push-rod” for rear. Skeleton of the car is made up of a 33.5 kg S275 steel tubular chassis and a simulated torsional rigidity of 2142 Nm / degree. To improve the dynamics of the vehicle at high speeds and lower weight, the full body is made of carbon fiber with bioresin and a complete aerodynamic package (front spoiler, rear spoiler and flat bottom with a low coefficient of Reynolds) generates 634 N of force against the ground at 80 km / h.

2.2.2. Powertrain

The battery pack or "Accumulator" has been designed to achieve the weight reduction levels set at the beginning of the season with a final configuration of 132 cells with a $LiCoO_2$ chemistry of 12500 mAh connected in series (132s1p), nominal final voltage obtained is 488.4 V and 6930 kWh of energy allow a peak discharge in the DC bus of 312 A.

Inventers are the BAMOCAR PG-D3 700/400 RS of the Unitek work in 4 quadrants and have two IGBT's Infineon FF600R12ME per phase. With 140 kW nominal power, incorporates a "cold-plate" liquid cooling that is capable of dissipating 2.4 kW of heat.

Regarding the engine, these are two permanent magnets synchronous working at 230 V AC with a maximum speed of 5000 rpm and 47 Nm nominal. The peak torque is in 130 Nm for 20 seconds and 40 Kw peak power.

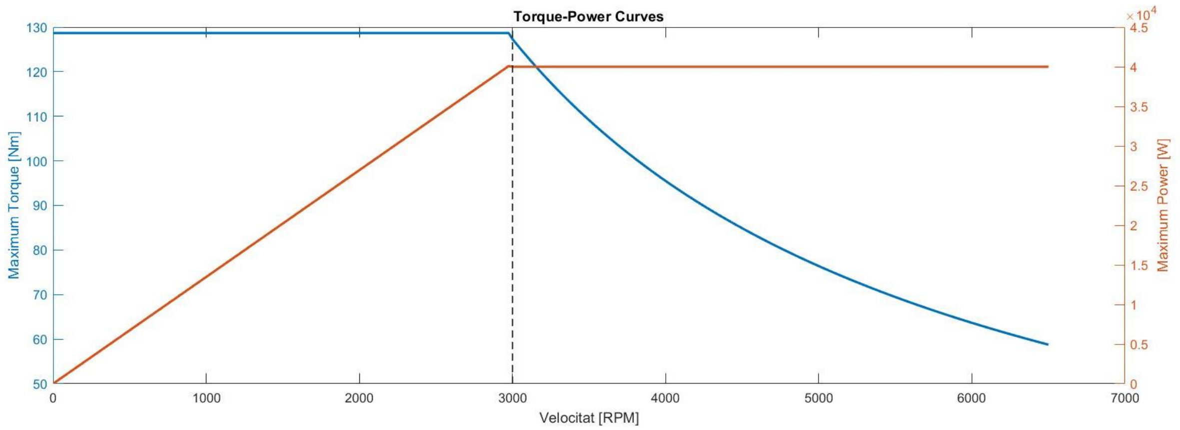

*Figure 7. Torque-Power Graph. **Source:** Vernis Motors.*

Finally, a pinion of 16 teeth of the motors is connected by a chain to a plate of 62 teeth with a multiplier ratio of 4.0625: 1 forming the transmission assembly. Bearings used are specific for Formula Student as well as the tripod joints. The Inverter-motor-transmission set is duplicated in order to achieve independent traction and control.

2.2.3. Sensors

Although in the algorithm these sensors are not used, the section is intended to show the system is feasible to bring it real, therefore sensors and its measured variables are proposed.

- Velocity sensor

Model	Cherry Sensors GS101205
Type	Hall Effect (IP67)
Output	Open Collector
Measured variable	V

Table 2. Characteristics velocity sensor.

- Suspension Sensor

Model	POSITEK LIPS P117
Type	LVDT (IP67)
Output	Analog 0.5 V – 9.5 V (75 mm)
Measured variable	$F_{z_{fl}}, F_{z_{fr}}, F_{z_{rl}}, F_{z_{rr}}$

Table 3. Characteristics suspension sensor.

- Accelerator and brake pedal sensor

Model	Race Technology Miniature Linear Movement Sensor
Type	LVDT (IP67)
Output	Analog 0-5 V (75 mm)
Resolution	Infinite
Measured variable	D_{comand}

Table 4. Characteristics pedal sensor.

- Steering Wheel sensor

Model	BEI Sensors 9360
Type	Hall effect (non-contact) (IP67)
Output	Analog 0.25 V – 4.75 V (270º)
Measured variable	δ

Table 5. Characteristics Steering Wheel sensor.

- Accelerometer and gyroscope

Model	Invensense MPU-6050
Type	Inertial mass
Output	Digital output
Measured variable	r, a_y

Table 6.Characteristics Accelerometer and gyro sensor .

- Motor encoder

Model	RLS RM44	
Type	Instrumental encoder with commutation	
Output	**Incremental**	**ABZ,A-B-Z-**
	Commutation	UVW
Measured variable	T_{rl}, T_{rr}	

Table 7. Characteristic sensor encoder.

3. Vehicle dynamics and mathematical model

In absence of a specific car simulator, the vehicle that takes part in this project is modelled with specific dimension and known parameters. From dynamic point of view, we cannot forget the driver influence within the car takes big part of the control and mostly how quickly will be. Even though, the behaviour of the driver is a concept that cannot be modelled. Shortly, will be commented that it is considered and a disturbance of the algorithm.

3.1. Study situation

In vehicle Dynamics modelling it is used a concept that helps the process form zero less complex. The "ladder of abstraction", introduced by [7], represents the steps since very basic model of vehicle to a large complex non-linear parameters automobile very close to reality. The so-called "bicycle model" is the first step of the ladder, is a simplified model with linear variables used to approximate the behaviour of the car.

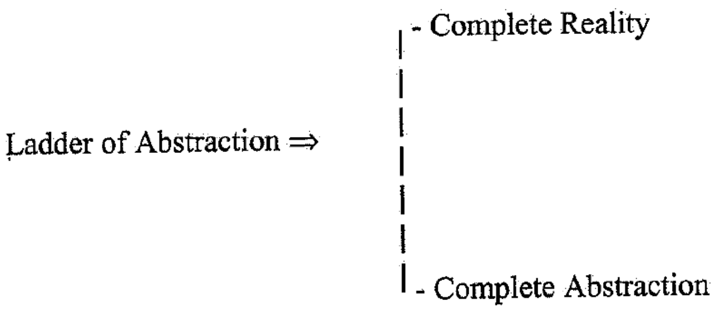

*Figure 8. "Ladder of Abstraction" concept. **Source:** [7]*

Consisting in two axes and single track vehicle, there are some considerations to take in account:

- No lateral longitudinal nor lateral load transfer
- Constant forward speed (set up by user)
- No aerodynamic effects
- No chassis or suspension compliance effects
- SAE axis convention (Figure 9)

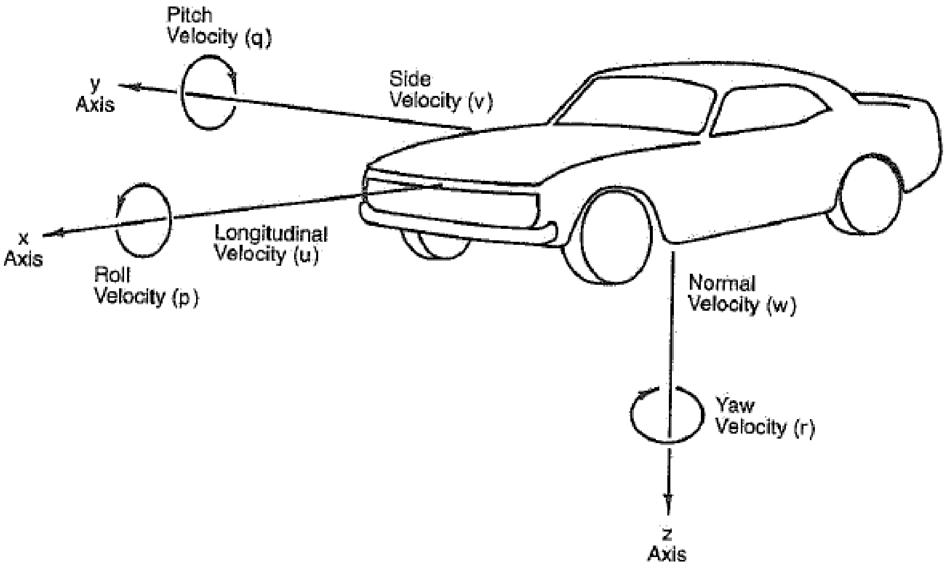

Figure 9. SAE Vehicle Axis System. ***Source:*** *[7].*

This model, also called two-degrees of freedom, has two output variables, β (attitude angle of the car) and r (yaw rate) and one input variable coming from driver, δ (steering angle) defined as the angle the front wheel is turning not the steering wheel angle.

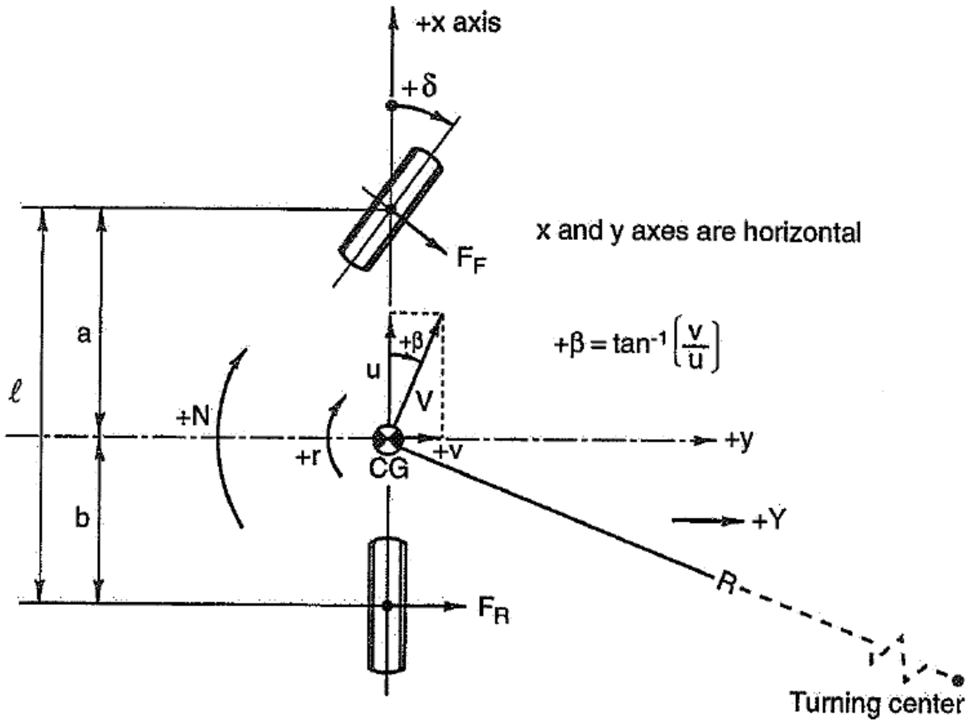

Figure 10. Bicycle model representation. ***Source:*** *[1]*

Normally, when the car is moving, we can differentiate in two phases. In straight line, no stability loses as the δ is zero, the outputs are also 0. However, when turning, δ is no longer 0, so 3 parts of the curve are defined divided in 2 state of the vehicle. Near the apex of the turn, we call "transient turn entry" when the variables mentioned before are changing with time, therefore the radius of the vehicle path is not constant.

When the δ starts to be constant and the radius of turning path too, the car is in steady-state phase. Finally, "turn exit" is defined as previous "turn entry". This division helps to imagine how is ideal cornering manoeuvre.

For a better understanding, in figure 11 is drawn the phases. Usually between "transient situation" (in green) and "steady-state" (blue) situation, the second one is most present in the curve. So, except in long continuous turn when there are no straight lines, this project will analyse steady-state situations.

Figure 11. Phases when turning. ***Source:*** *Own*

3.2. Steady-state

If a Picture is taken when the automobile is just in steady-state, for *R* and *V* constant, lateral force due lateral acceleration cannot be neglected and tires try to keep the car in its path, so a force analysis can be done in CG and 3 different stability situation are discriminated: neutral-steer, under-steer and over-steer.

First one assume that CG is middle wheelbase situated (means that a t and b are equal) and tires have the same pressure and same Cornering Stiffness value. Equilibrium force equations are the next presented.

$$m \cdot a_y = F_{y_f} + F_{y_r} \tag{3.1}$$

$$m \cdot \frac{V^2}{R} = C_f \cdot \alpha_f + C_r \cdot \alpha_r \tag{3.2}$$

And equilibrium moment around CG is:

$$\sum N = 0 = F_{y_f} \cdot a + F_{y_r} \cdot b \tag{3.3}$$

$$C_f \cdot \alpha_f \cdot a = C_r \cdot \alpha_r \cdot b \tag{3.4}$$

Force Generation from tire is the result of the slip angle and Cornering Stiffness. In order to define briefly this concepts, slip angle is the angle between the plane in X axis of the tire and the road where the car is rolling when turning, and Cornering Stiffness is the concept that relates the force of the tire and slip angle. From equations 3.4, for same distance a and b, and C_f C_r, one can see the same steer angle is required to negotiate the same radious curve regardless the speed. In magnitude, the rear shaft steers the vehicle the same as the front.

$$\delta = \frac{l}{R} \tag{3.5}$$

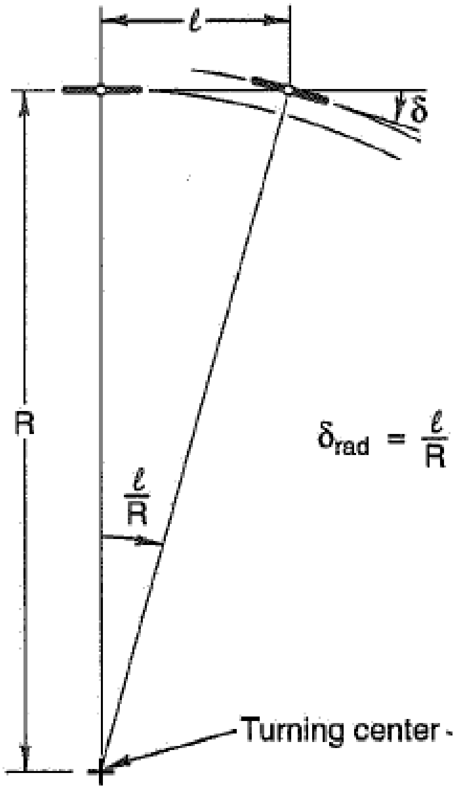

Figure 12. Geometry of vehicle when turning. ***Source:*** *[7]*

Figure 12 shows the second case, under-steer, for same δ, the CG is located at $^1/_3$ wheelbase from front shaft. The static load now is greater in front and more force has to be invest to beat the inertial component due lateral force. If the Cornering Stiffness remain the same, the slip angles change the same percentage as the rear and front loads, so a greater front slip angle and smaller rear slip angle will appear in the analysis.

$$b = 2 \cdot a \tag{3.6}$$

$$\alpha_f = 2 \cdot \alpha_r \tag{3.7}$$

We say the vehicle is understeering when for a radius curve *R,* not enough steering angle is applied to maintain the path of the road. Feels like the vehicle want to go out the desired path. To avoid this situation, the equation 3.8 must be followed for δ.

$$\delta = \frac{l}{R} + (-\alpha_f + \alpha_r) \tag{3.8}$$

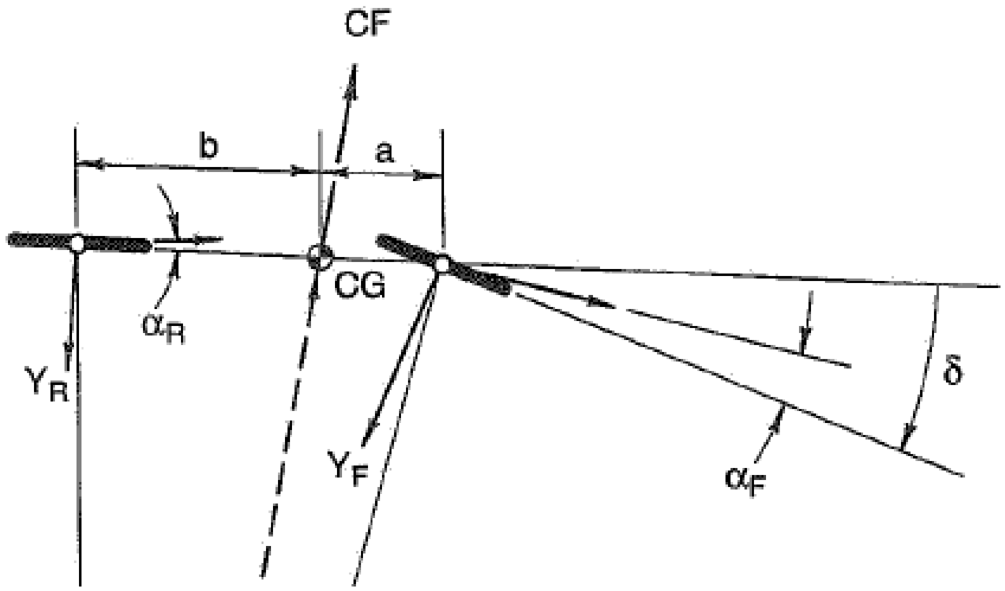

Figure 13. Understeer geometry. ***Source:*** *[7]*

On the contrary, now the CG is located at $^1/_3$ on the rear side, in figure 13 can be seen now the loads distribution. The rear part will carry twice the front therefore this one will be the dominant when turning. The force generated by rear slip angle will steer the car reducing too much the curve radius path of the vehicle. We say the vehicle is over-steering when too much steering angle is applied and suddenly the vehicle "spin". For easy comparison in racing, drifting discipline stay constantly in over-steering situation to negotiate the circuit.

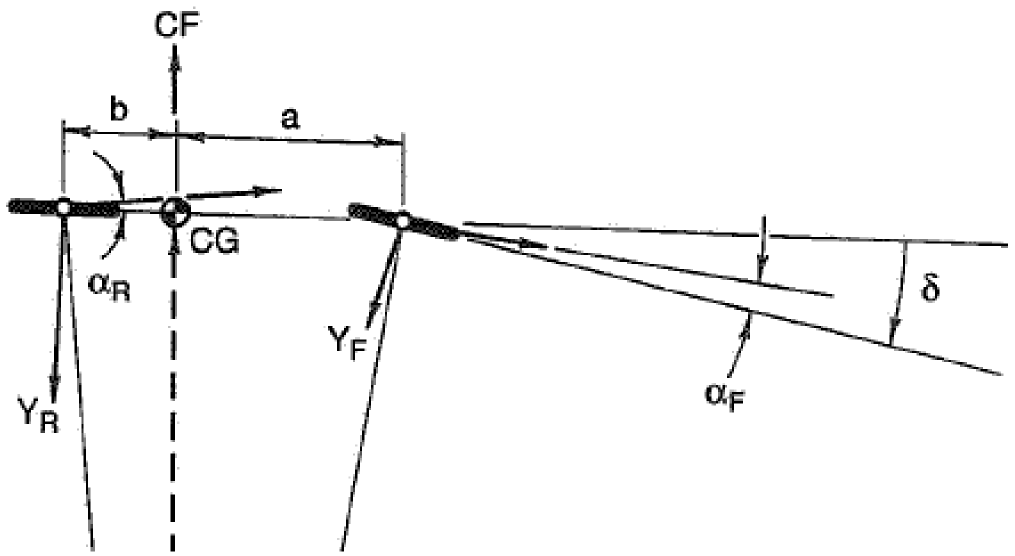

Figure 14. Oversteer geometry. ***Source:*** *[7]*

For summarize this definition, in neutral-steer, no δ has to be changed, the vehicle is estable. When under-steer the driver has to increase the δ input to correct an instability situation, and finally in over-steering, δ has to be reduced to return in neutral-steer.

3.3. Equation of motion

The thesis bases its objectives in the model of the car as transfer function to understand how reacts for a certain input. With the two degrees of freedom model presented before, in this chapter will be developed the equations which enables the calculation of the variables of interest from forces and moments experimented by the vehicle. The yaw rate r, or how quickly the car is turning around its Z axis, with this variable it can be defined how much has to steer the car to follow the curve is passing. And the side slip angle β, the angle between the wheelbase of the car and the velocity vector, is useful to know how instable is the car due under-steer or over-steer situation.

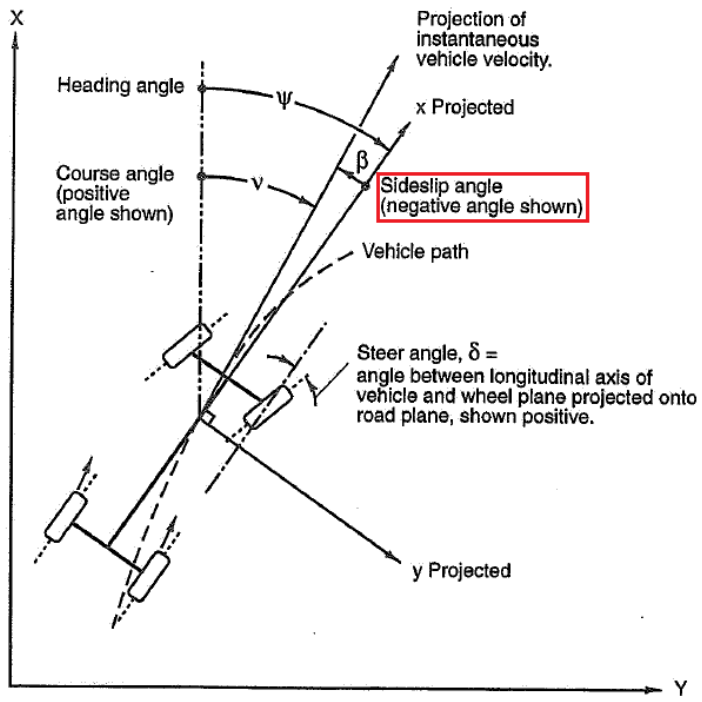

Figure 15. Sideslip angle. ***Source:*** *[7]*

From Newton's second law:

$$T = I \cdot \alpha \ (where\ \alpha = angular\ acceleration) \tag{3.9}$$

$$F = m \cdot a \tag{3.10}$$

We transform it to most comfy demonstration in:

$$M_z = I_z \cdot \frac{dr}{dt} \tag{3.11}$$

$$F_y = m \cdot a_y \tag{3.12}$$

M_z i F_y represents the yaw moment and lateral force of the vehicle evironment. In 3.11 and 3.12, both have inertial components; the I_z as inertia moment in Z axis and m the mass of the vehicle, the rest of equation represents the angular acceleration and lateral acceleration. The last one can be disaggregate in two parts. When turning the velocity V vector is formed by the x component, u ,and the y component, v, but as the angle formed between is so small from now on V will be u. Assuming this, in equation 3.13, V is multiplied by the angular velocity r and the lateral velocity component to the acceleration is added.

$$a_y = V \cdot r + \dot{v} = V \cdot r + V \cdot \dot{\beta} = V(r + \dot{\beta}) \tag{3.13}$$

As this model effects of mass transfers and aerodynamic effects were not taken into account, it is included the slip angle of the tires as a product of the forces and final lateral moments. These, as we shall see, are a function in themselves of the variables of interest that we have mentioned before.

$$\alpha_f = \frac{v + a \cdot r}{V} - \delta = \frac{v}{V} + \frac{a \cdot r}{V} - \delta = \beta + \frac{a \cdot r}{V} - \delta \tag{3.14}$$

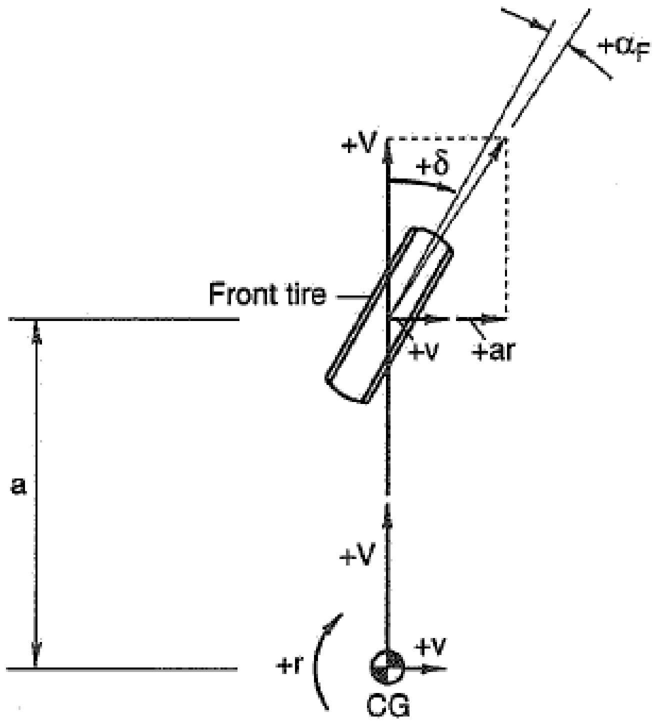

Figure 15. Front Slip angle. ***Source:*** *[7]*

$$\alpha_r = \frac{v - b \cdot r}{V} = \frac{v}{V} - \frac{b \cdot r}{V} = \beta - \frac{b \cdot r}{V} \qquad (3.15)$$

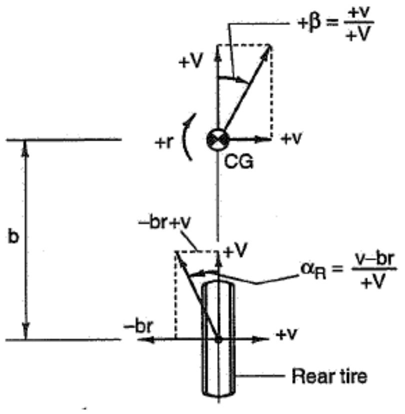

Figure 16. Rear slip angle. ***Source:*** *[7]*

If the force and moment equilibrium is considered, the result is*:*

$$F_y = F_{y_f} + F_{y_r} = C_f \cdot \alpha_f + C_r \cdot \alpha_r \qquad (3.14)$$

$$M_z = M_{z_f} + M_{z_r} = C_f \cdot \alpha_f \cdot a + C_r \cdot \alpha_r \cdot b \qquad (3.15)$$

Thus:

$$F_{y_f} = C_f \cdot \left(\beta + \frac{a \cdot r}{V} - \delta\right) = C_f \cdot \beta + C_f \frac{a \cdot r}{V} - C_f \delta \qquad (3.16)$$

$$F_{y_r} = C_r \left(\beta - \frac{b \cdot r}{V}\right) = C_r \cdot \beta - C_r \frac{b \cdot r}{V} \qquad (3.17)$$

Arranging terms of force and moments:

$$F_y = F_{y_f} + F_{y_r} = C_f \cdot \beta + C_f \frac{a \cdot r}{V} - C_f \cdot \delta + C_r \cdot \beta - C_r \frac{b \cdot r}{V} \qquad (3.18)$$

$$M_z = M_{z_f} + M_{z_r} = F_{y_f} \cdot a + F_{y_r} \cdot b \tag{3.19}$$

$$M_z = C_f \cdot \beta \cdot a + C_f \frac{a^2 \cdot r}{V} - C_f \cdot \delta \cdot a + C_r \cdot \beta \cdot b - C_r \frac{b^2 \cdot r}{V} \tag{3.20}$$

$$M_z = \left(a \cdot C_f + b \cdot C_r\right)\beta + \frac{1}{V}\left(a^2 \cdot C_f + b^2 \cdot C_r\right)r + a \cdot C_f \cdot \delta \tag{3.21}$$

Lastly, combining 3.11,3.12, 3.13 and 3.20, 3.23, the so-called equations of motion are defined.

$$m \cdot V(r + \dot{\beta}) = \left(C_f + C_r\right)\beta + \frac{1}{V}\left(a \cdot C_f - b \cdot C_r\right)r - C_f\delta \tag{3.22}$$

$$I_z \cdot \dot{r} = \left(a \cdot C_f + b \cdot C_r\right)\beta + \frac{1}{V}\left(a^2 \cdot C_f + b^2 \cdot C_r\right)r + a \cdot C_f \cdot \delta \tag{3.23}$$

3.4. Stability factor

Even though the ideal situation is NS, oscillations between this point exist, bringing the car to OS or US. The stability factor measures how long from NS the car is, and this is used in the algorithm to compensate the aim to be in NS. Presented in [1] the stability factor K is the factor in the equation 3.26, that relates the steering input δ with the desired yaw rate r.

$$\frac{r}{\delta} = \frac{V/_l}{1 + K \cdot V^2} \tag{3.24}$$

K takes in account the Cornering Stiffness of the tires and the front and rear shafts distances from the CG.

$$K = \frac{m}{l}\left(\frac{{aC_f - bC_r}/{-(C_f + C_r)}}{-aC_f - \left({aC_f - bC_r}/{C_f + C_r}\right)(-C_f)}\right) \tag{3.25}$$

According to SAE convention, when K is positive the vehicle will be in US and on the contrary, when K is negative the car will be in OS. Hypothetically, if $K = 0$ (meaning no US or OV), the equation 3.26 reduces to the one presented in 3.28, if divided by the V , the path curvature is obtained. 3.29 can be compared with 3.5 and a good conclusion is when NS the steering value has no dependence with the velocity taken in a curve.

$$\frac{r}{\delta} = \frac{V}{l} \tag{3.26}$$

$$\frac{1/_R}{\delta} = \frac{1}{l} \tag{3.27}$$

4. Control Structure

Divided in different levels, the algorithm can be more easily understood. Each level represent how far are the variables from a real output, meaning that the output of high level controller is not the same that the output of low level controller, which are the final torque commands will be sent to inverters. From first, this control loop starts from the driver with the yaw reference generator block, through steering wheel, steers front wheels and generate a desired yaw rate.

The high level controller is key to obtain good results. In this block is included a PI controller and the tuning method. Its output is the moment around Z axis of car needed to turn the car the amount the driver wants. Medium and Low controller blocks are in charge to transform this M_z in differential torque to rear wheels. Constraints to limit output power are applied to protect motor physical limits.

In negative feedback appears the modelled vehicle plant. It represents the "sensor" that returns the real wheel torques to actual yaw rate.

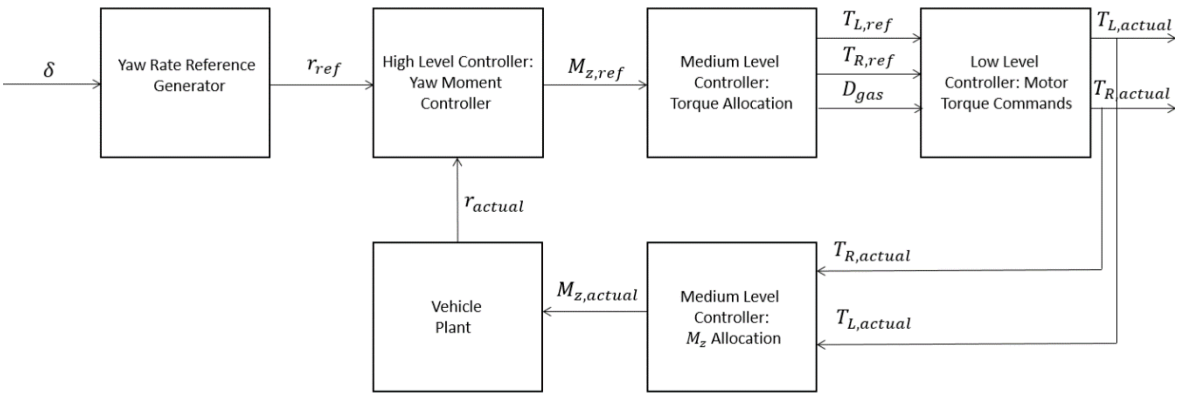

*Figure 16. General structure of the control system. **Source:** Own*

4.1. Yaw Reference Generator

From steering wheel, mechanically connected to front shaft with a pinion-rack system as figure 19 shows, the driver steers front wheels with δ angle.

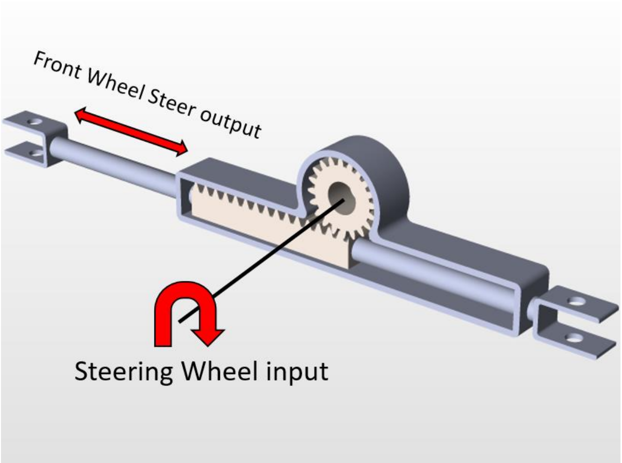

*Figure 17. Pinion-Rack gearing. **Source:** e-Tech Racing*

Commented in the introduction of the chapter, this block is in charge to generate how much will the driver turn in terms of yawing velocity as a reference r_{des} . Using the equation 3.26, can be seen that depends of the steady-state situation. If we imagine a OS moment when turning, and the driver wants to steer even more, it is not logic to keep or apply even more differential torques. So the stability factor K will correct the situation bringing the car to NS state.

$$\frac{r_{des}}{\delta} = \frac{V/_l}{1 + KV^2}$$

4.2. Vehicle Plant

A control with negative feedback loop strategy is chosen, the negative input to the add-subtract block comes from the vehicle plant. The real behaviour of the car is described with the motion equations which appear in Chapter 3. So transfer function is developed in order to transform the actual M_z to the actual r.

$$I_z \cdot \dot{r} = \left(aC_f + bC_r\right)\beta + \frac{1}{V}\left(a^2C_f + b^2C_r\right)r + aC_f\delta \tag{4.1}$$

$$m \cdot V(r + \dot{\beta}) = \left(C_f + C_r\right)\beta + \frac{1}{V}\left(aC_f - bC_r\right)r - C_f\delta \tag{4.2}$$

Inputs and outputs of the algorithm are linear and work in time domain, but extensively known in control theory, in order to arrange terms of equation and tune any control strategy, it is better to transform equations in Laplace domain.

$$I_z \cdot r \cdot s = \left(aC_f + bC_r\right)\beta + \frac{1}{V}\left(a^2C_f + b^2C_r\right)r + aC_f\delta \tag{4.3}$$

$$m \cdot V(r + \beta \cdot s) = \left(C_f + C_r\right)\beta + \frac{1}{V}\left(aC_f - bC_r\right)r - C_f\delta \tag{4.4}$$

For the yaw controller, one can see δ is not a controllable variable beacause it is a human source, so it is treated as a disturbance. The loop is closed with the real M_z provided by output torque. Including this last variable and rearranging equations, real yawing velocity can be isolated .

$$r \cdot (I_z \cdot s - \frac{1}{V}\left(a^2C_f + b^2C_r\right)) = \left(aC_f + bC_r\right)\beta + M_z \tag{4.5}$$

$$\beta \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right) = r \cdot (\frac{1}{V}\left(aC_f - bC_r\right) - m \cdot V) \tag{4.6}$$

Following:

$$\frac{r \cdot (I_z \cdot s - \frac{1}{V}\left(a^2C_f + b^2C_r\right)) - M_z}{\left(aC_f + bC_r\right)} = \beta \tag{4.7}$$

$$\beta = \frac{r \cdot (\frac{1}{V}\left(aC_f - bC_r\right) - m \cdot V)}{\left(m \cdot V \cdot s - \left(C_f + C_r\right)\right)} \tag{4.8}$$

$$\frac{r \cdot (I_z \cdot s - \frac{1}{V}\left(a^2 C_f + b^2 C_r\right)) - M_z}{\left(aC_f + bC_r\right)} = \frac{r \cdot (\frac{1}{V}\left(aC_f - bC_r\right) - m \cdot V)}{\left(m \cdot V \cdot s - \left(C_f + C_r\right)\right)} \tag{4.9}$$

$$\left(r \cdot \left((I_z \cdot s - \frac{1}{V}\left(a^2 C_f + b^2 C_r\right))\right) - M_z\right) \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right) = r \cdot \left((\frac{1}{V}\left(aC_f - bC_r\right) - m \cdot V)\right) \cdot \left(aC_f + bC_r\right) \tag{4.10}$$

$$\begin{aligned} &\left(r \cdot \left((I_z \cdot s - \frac{1}{V}\left(a^2 C_f + b^2 C_r\right))\right)\right) \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right) - M_z \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right) \\ &\qquad = r \cdot \left((\frac{1}{V}\left(aC_f - bC_r\right)) \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right) - m \cdot V \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right)\right) \end{aligned} \tag{4.11}$$

$$\begin{aligned} r&\left(\left((I_z \cdot s - \frac{1}{V}\left(a^2 C_f + b^2 C_r\right))\right) \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right)\right. \\ &\qquad \left. - \left((\frac{1}{V}\left(aC_f - bC_r\right)) \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right) - m \cdot V \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right)\right)\right) \\ &\qquad = M_z \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right) \end{aligned} \tag{4.12}$$

$$\begin{aligned} &\frac{r}{M_z} \\ &= \frac{\left(m \cdot V \cdot s - \left(C_f + C_r\right)\right)}{\left((I_z \cdot s - \frac{1}{V}\left(a^2 C_f + b^2 C_r\right))\right) \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right) - \left((\frac{1}{V}\left(aC_f - bC_r\right)) \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right) - m \cdot V \cdot \left(m \cdot V \cdot s - \left(C_f + C_r\right)\right)\right)} \end{aligned} \tag{4.13}$$

The final transfer function is:

$$\frac{r}{M_z} = \frac{(m \cdot V)s - (C_f + C_r)}{(m \cdot V \cdot I_z) \cdot s^2 - \left(m \cdot V \cdot \frac{1}{V}(a^2C_f + b^2C_r) + (C_f + C_r) \cdot I_z\right) \cdot s + \left((C_f + C_r) \cdot \frac{1}{V}(a^2C_f + b^2C_r) - (aC_f - bC_r) \cdot \left(\frac{1}{V}(aC_f - bC_r)\right) + m \cdot V\right.} \tag{4.14}$$

Different letters have been assigned to each term:

$$\frac{r}{M_z} = \frac{A \cdot s - B}{C \cdot s^2 - D \cdot s + E} \tag{4.15}$$

Being:

$$A = (m \cdot V)$$

$$B = (C_f + C_r)$$

$$C = (m \cdot V \cdot I_z)$$

$$D = \left(m \cdot V \cdot \frac{1}{V}(a^2C_f + b^2C_r) + (C_f + C_r) \cdot I_z\right)$$

$$E = \left((C_f + C_r) \cdot \frac{1}{V}(a^2C_f + b^2C_r) - (aC_f - bC_r) \cdot \left(\frac{1}{V}(aC_f - bC_r)\right) + m \cdot V \cdot (aC_f - bC_r)\right)$$

4.3. High Level Controller

This block describes the analysis of the vehicle plant and the proposed PI yaw velocity control for the vehicle. The main idea of the controller is to manage the coming set point input from yaw reference generator r_{des} and compare it with the feedback provided by the actual yawing velocity r_{act}, after processing the error, the output will be a quick M_z response.

To do that, first it is needed to know how stable is the plant in open loop and a pole-zero map and step function is plotted.

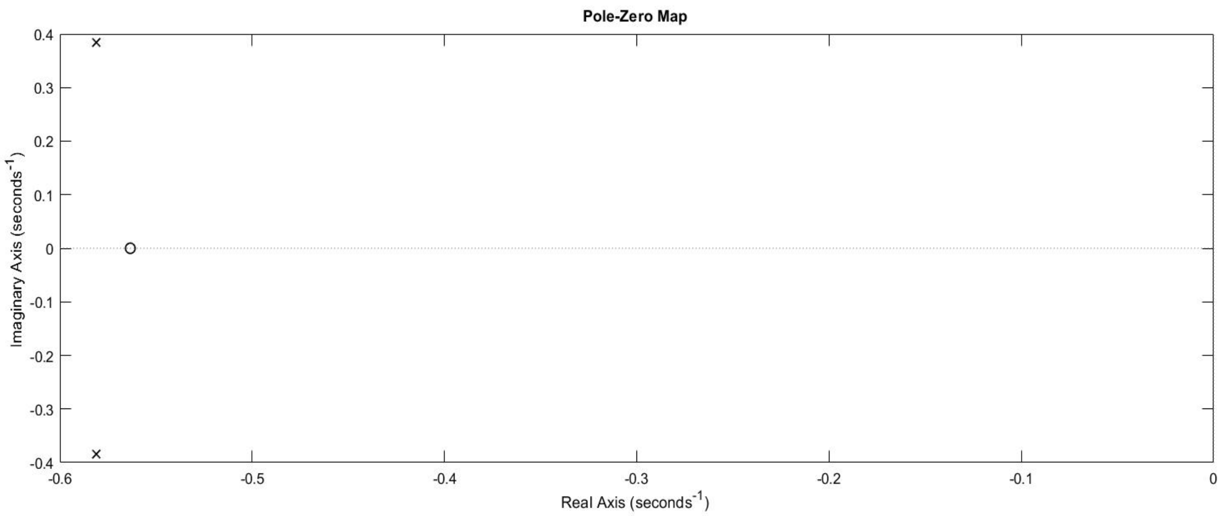

Figure 18. Pole-zero map. ***Source:*** *Own*

Located in:

$$\times \qquad s = -0.5810 + 0.3838j$$
$$\times \qquad s = -0.5810 - 0.3838j$$
$$\circ \qquad s = -0.5634$$

And the step response is:

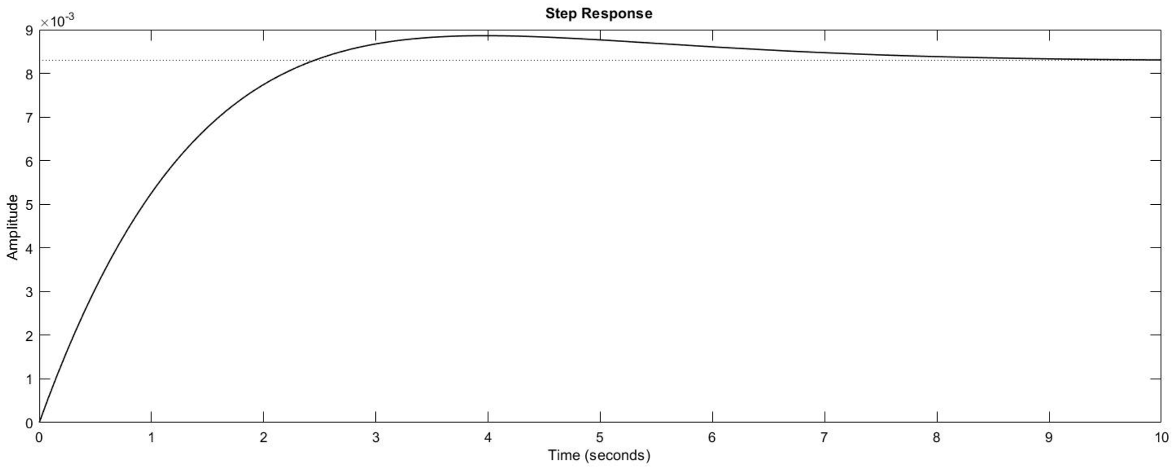

Figure 19. Plant step response. ***Source: Own***

In figures 18 and 19 can be seen the plant is stable but with a rise time of 1.71 seconds, an overshoot of 6.77 % and stabilization time of 7.06 seconds this system is clearly not acceptable for the yaw controller since the delay form input to output has to be less than half second. Hence a design parameters have been proposed to improve the system behaviour. With a 0.2 seconds of rise time and a 5 % of overshoot with a maximum time of stable setting of 0.3 seconds as a goal to reach.

Previously to present the controller and its tuning, it is needed to transform the controlled variable output to the input for vehicle plant, a Moment in Z axis. So the equation 4.16 is used.

$$M_z(t) = \dot{r}_{controled}\ (t) \cdot I_z \tag{4.16}$$

Where $\dot{r}_{controled}\ (t)$ is the yaw acceleration and I_z the inertia moment of the car in Z axis. If we work in Laplace domain:

$$M_z(s) = r_{controled}(s) \cdot I_z \cdot s \tag{4.17}$$

We have a clearly derivative term in our loop

$$\frac{M_z(s)}{r_{controled}(s)} = I_z \cdot s \tag{4.18}$$

Going deep in the tuning terms of control, the scheme high level controller is build and the transfer function in calculated, renaming the blocs in R(s) as a vehicle Plant, C(s) as PI controller and G(s) as the derivative part.

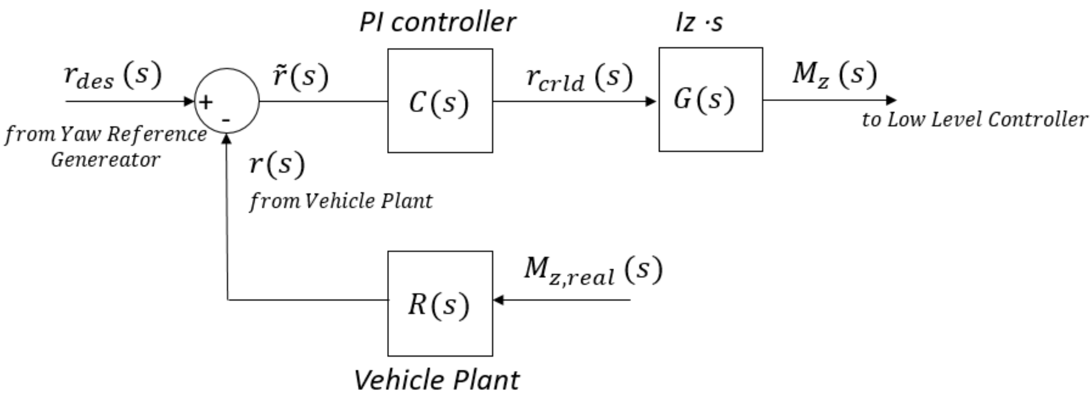

Figure 20. High Level Controller. ***Source:*** *Own*

The closed feedback loop equation:

$$T\ (s) = \frac{C(s)\cdot G(s)}{1 + C(s)\cdot G(s)\cdot R(s)} \tag{4.19}$$

The proportional and integrative gains $\mathrm{K_p}, \mathrm{K_i}$ can be adjusted by root locus methodolody. Taking the characteristic equation and proceed the with the initial values of the plant, $\mathrm{K_p}$ can be fixed at a reasonable value and sweep with $\mathrm{K_i}$. An interative process has been taken to find the most $\mathrm{K_p}, \mathrm{K_i}$ suitable for the controller.

$$0 = 1 + C(s)\cdot G(s)\cdot R(s) \tag{4.20}$$

$$0 = \left(\frac{K_p\cdot s + K_i}{s}\right)\cdot(140\cdot s)\cdot\left(\frac{(17{,}01\cdot s + 44{,}44)\cdot 10^{-4}}{s^2 + 5{,}41\cdot s + 4{,}19}\right) \tag{4.21}$$

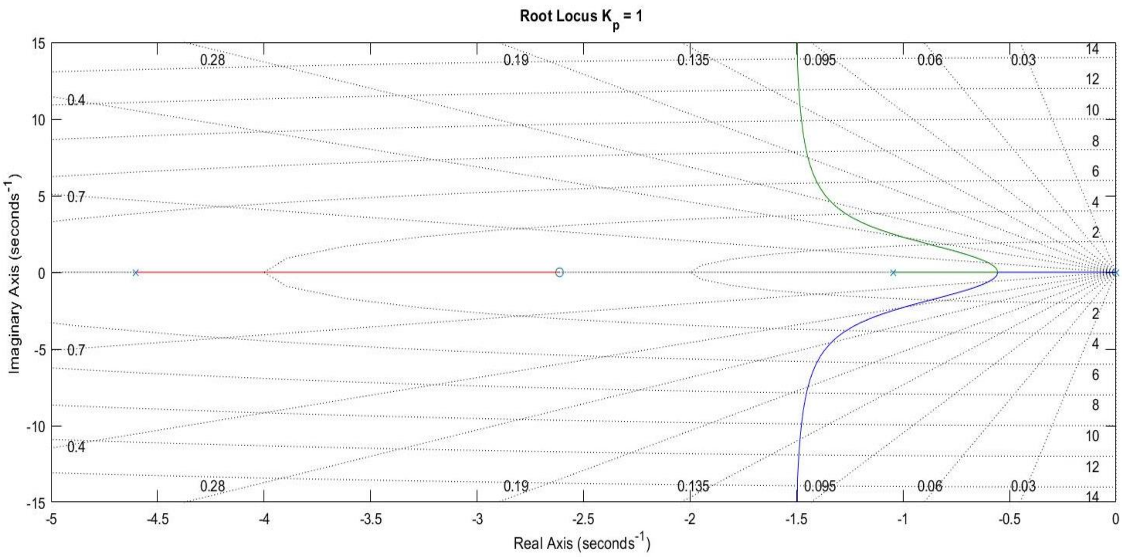

*Figure 21. Root locus map Closed-Loop. **Source:** Own*

For an assigned K_p = 1, a more than enough good result is found with $K_i = 100$

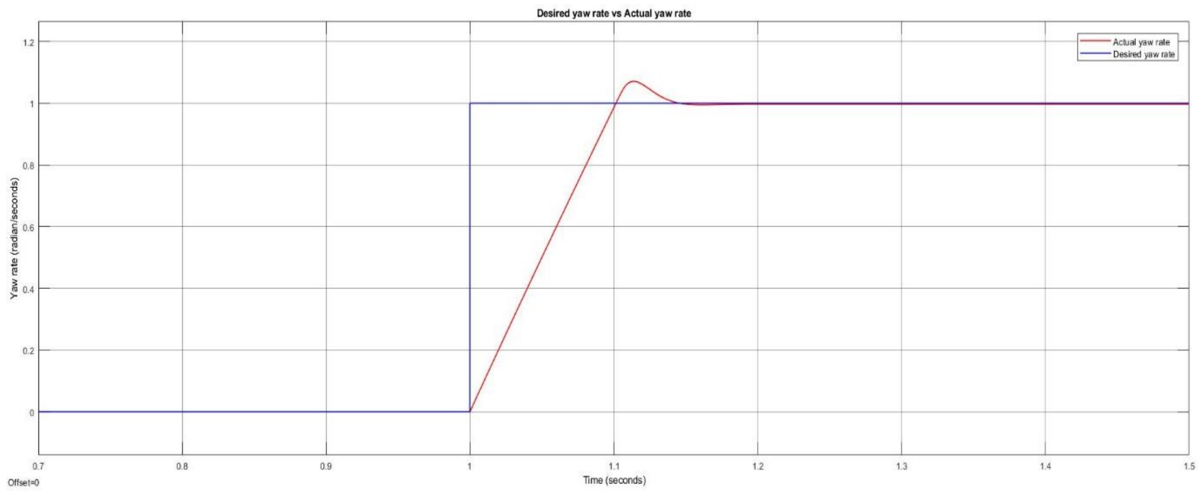

Figure 22. Step Response Closed Loop.

Can be compared the step response of the figure 19 and the figure 22, the new one with an 80.92 milliseconds of rise time and 6.98 % of overshoot is clearly a big improvement in system quick response and the values are inside the requirements of design.

4.4. Medium Level Controller

Summarizing this block converts $\mathrm{M_z}$ in torque to the motors using purely distances (d), moments or torques (T) and forces (F).

$$T = F \cdot d \tag{4.22}$$

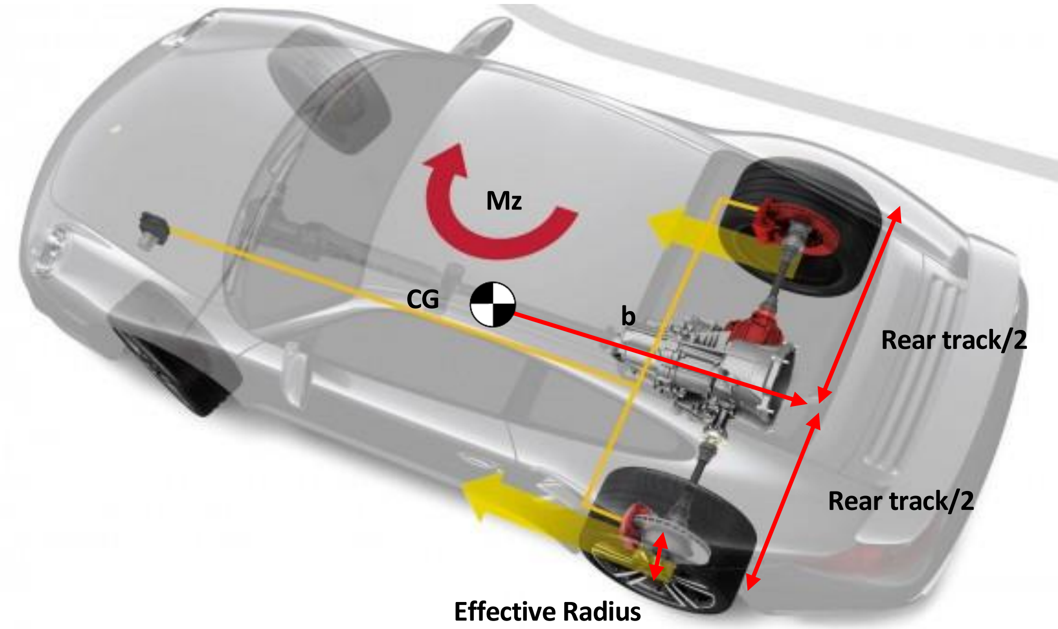

Figure 23. Force and distance distribution on the rear part. ***Source:*** *Own*

The equation 4.22 changes the value of M_z to longitudinal forces of the tires F_x. In equation 4.23 reflects the same value of force but with different sign, essentially, the differential pair. The figure 23 shows both yellow arrows forward meaning that never will be a negative value of torque, the same torque set point is sent to motors but added or substrate from it made the calculation.

$$M_z = F_x \cdot t_r \tag{4.23}$$

$$M_z = F_{x_{wl}} \cdot \frac{t_r}{2} - (-F_{x_{wr}} \cdot \frac{t_r}{2}) \tag{4.24}$$

When longitudinal forces of each tire are isolated, the torques in the motor need to be allocated. To know what torque is transmitted to rear bearing, the effective radius of the tire is used. This radius is not the wheel radius but the measure taken in static situation when the driver is within the car at 0.8 bar of pressure at the tires.

$$F_x = {T_{Wheels}}/{r_{eff}} \tag{4.25}$$

Finally, the torque of motors comes through rear bearings to transmission system with a ratio of 4.025.

$$T_{Wheels} = r_{trans} \cdot T_{motors} \tag{4.26}$$

$$T_{motors} = T_{wl} = -T_{wr} \tag{4.27}$$

During the different race situations there will also be straight sections, the algorithm starts with an initial common torque T_n that will be shared with both motors, the differential calculated in equation 4.27 will add and subtract depending of the high level controller output.

The last big step of this block is the gain of these torques. The accelerator pedal will control the output with a multiplier factor $D_{command}$ between in 0 and 1.

$$T_{lt} = D_{command} \cdot (T_n + T_{wl}) \tag{4.28}$$

$$T_{rt} = D_{command} \cdot (T_n + T_{wr}) \tag{4.29}$$

As it will appear in the low level controller, the M_z input vehicle plant will receive is not the output M_z of the high level controller, limit constraints appear in the middle so the way back starting for the real torques has to be done. The Moment allocator block uses the same equations 4.23. The inverse development is:

$$T_{wl} = T_l \cdot r_{trans} \tag{4.30}$$

$$T_{wr} = T_r \cdot r_{trans} \tag{4.31}$$

$$F_{xr} = {}^{T_{wr}}\!/_{r_{eff}} \tag{4.32}$$

$$F_{xl} = {}^{T_{wl}}\!/_{r_{eff}} \tag{4.33}$$

$$M_z = F_{xl} \cdot \frac{t_r}{2} + F_{xr} \cdot \frac{t_r}{2} \tag{4.34}$$

4.5. Low Level Controller

Since the physical construction of motors has power limits, in order to prevent damage to them, due the calculation of the previous block (high values of slew rate demand) can lead to the delivery of torques that overload the capacity of the motors. Therefore, a look-up table Max Torque - Rpm has been inserted before the torque command are sent to inverters as a constraint.

$$T_l = min\begin{cases} D_{command} \cdot (T_n + T_{wl}) \\ Motor\ Maximum\ Torque \end{cases} \tag{4.35}$$

$$T_r = min\begin{cases} D_{command} \cdot (T_n + T_{wr}) \\ Motor\ Maximum\ Torque \end{cases} \tag{4.36}$$

5. Controller Results

For simulations, Matlab-Simulink software has been used. According to [1] and [8] different inputs has been used in order to validate the functionality of the algorithm: the rapid response of the yaw rate, and the output torques.

To set common test values for all scenarios, the open variables δ and V has been set to 0.4 radians and 11 m/s. The initial torque T_n will be 80 Nm and $D_{command}$ will be 1.

5.1. Step input

An unrealistic situation in racing, this discontinuous function is useful to see how the plant responds and adjust tuning parameters. Figure 24 and figure 25 show respectively the tracking of the yaw rate and the torques curves at the exit of low level controller.

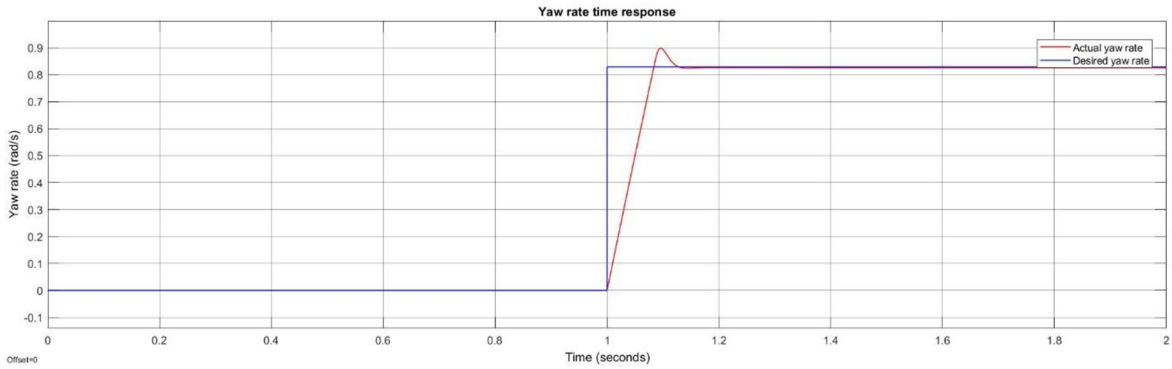

Figure 24. Yaw rate time response with step input

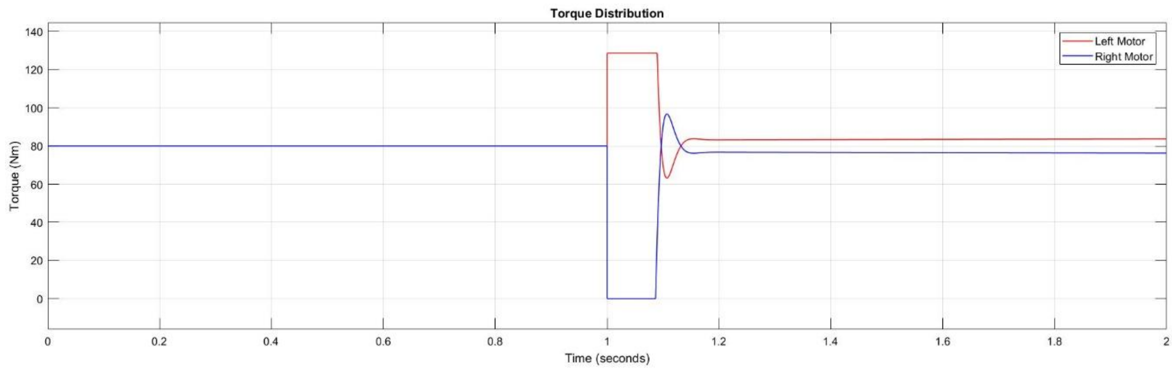

Figure 25. Torque Response to step input

Regarding the torques, one can see that they saturate at 0.001 seconds at the moment of the step, the left wheel with positive values is the result the look-up table constraint of the last block as a protection, the right motors stops completely to 0 torque delivery. This is reasonable since an elevate clockwise yaw moment is needed to follow the yaw rate reference. When the system stabilizes, torques fall down to normal values.

5.2. Ramp input

Like the previous case, this is an approximation of J-turn maneuver but softer that step. Still far from real case due sharp discontinuities in slope.

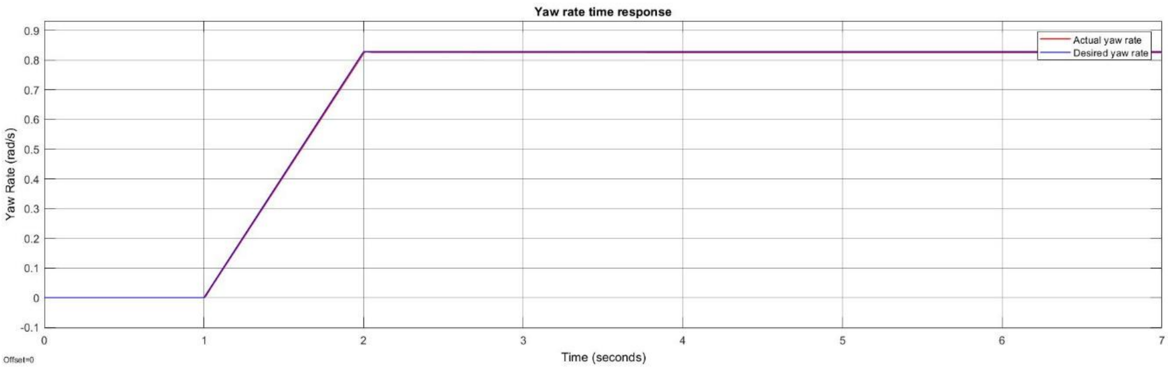

Figure 26. Yaw rate time response with step input

As the step response, this one seems to be pretty good, because there is almost exact desired yaw rate following. In figure 27, can be seen the detail where a little delay of 8 milliseconds and relative error of 3.3% exists.

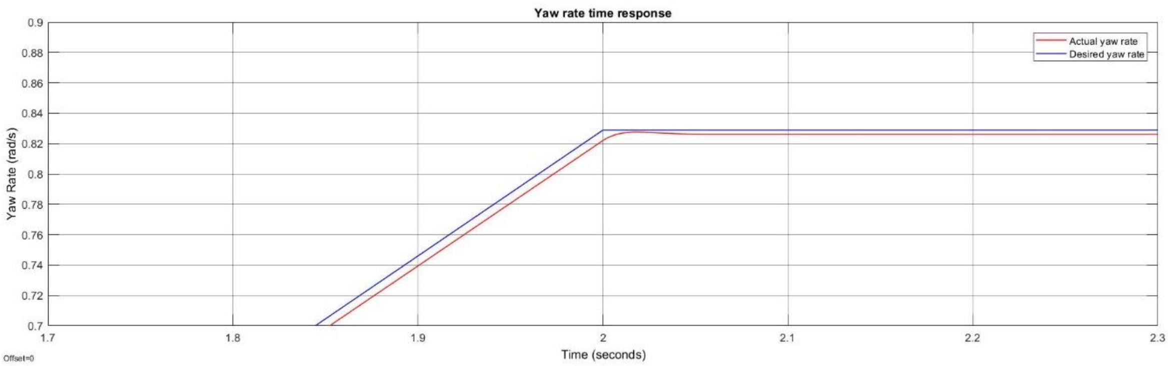

Figure 27. Step input detail

Regarding the output torques, there is no more limitation and we see the discontinuities are solved very quickly. The only tricky thing can be observed is a 4 second complete stabilization torques. This can be solved adjusting the K_i to grater values.

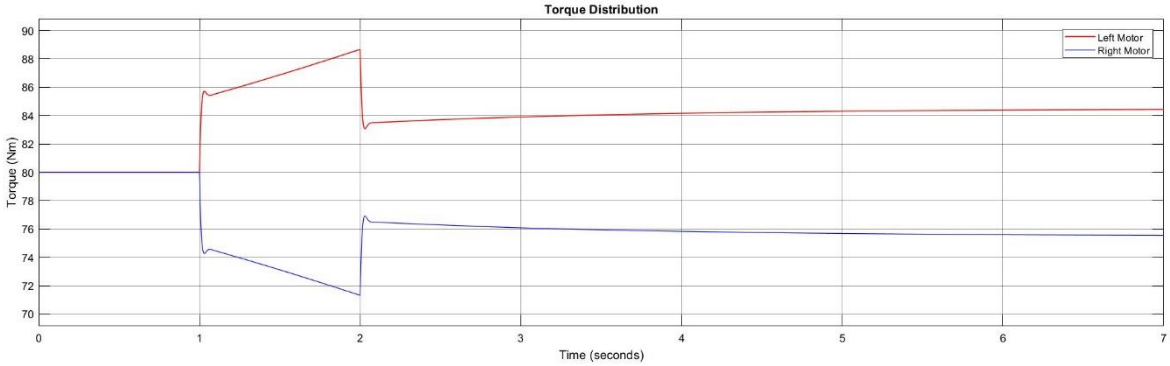

Figure 28. Torque response to ramp input

5.3 J-Turn

More realistic scenario is J-Turn trajectory due there are no more discontinuities in the input. This one consists in steer the vehicle the way it changes its direction 180 degrees around constant radius curve coming from straight line. In figure 29, can be appreciated the start position and end position and the road path the vehicle has to follow.

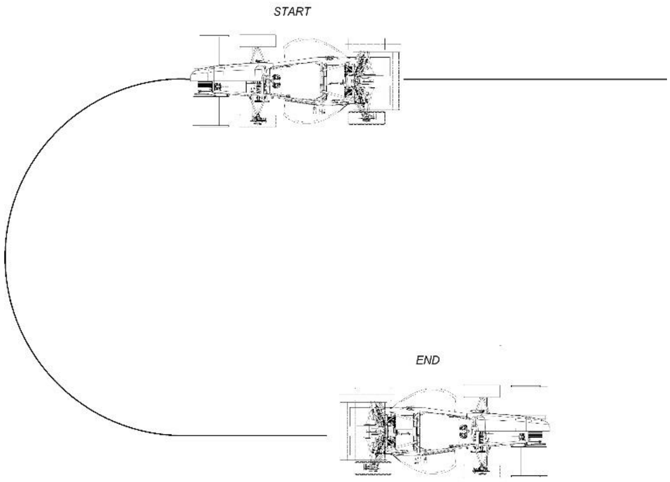

Figure 29. J-turn maneuver. ***Source:*** *Own*

To model this input in simulation has been used the arctan function like [1] suggest. The output response is showed in figure 30 and 31.

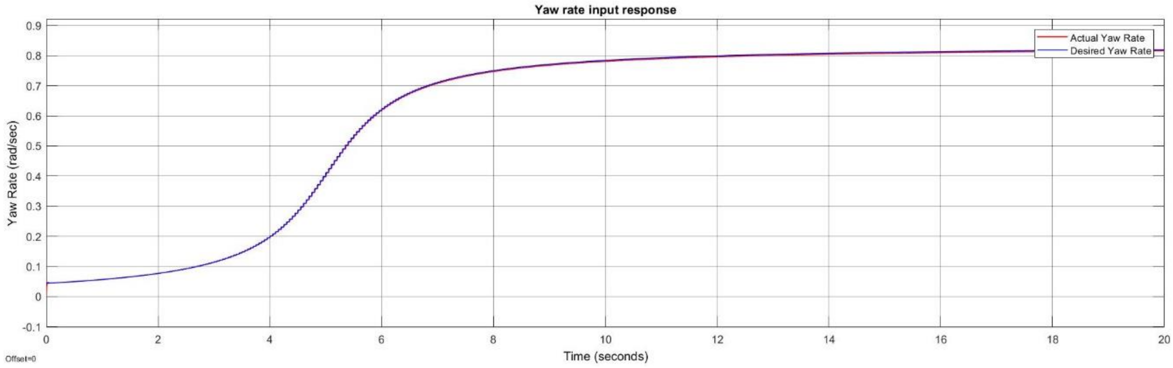

Figure 30. J-turn input yaw rate response

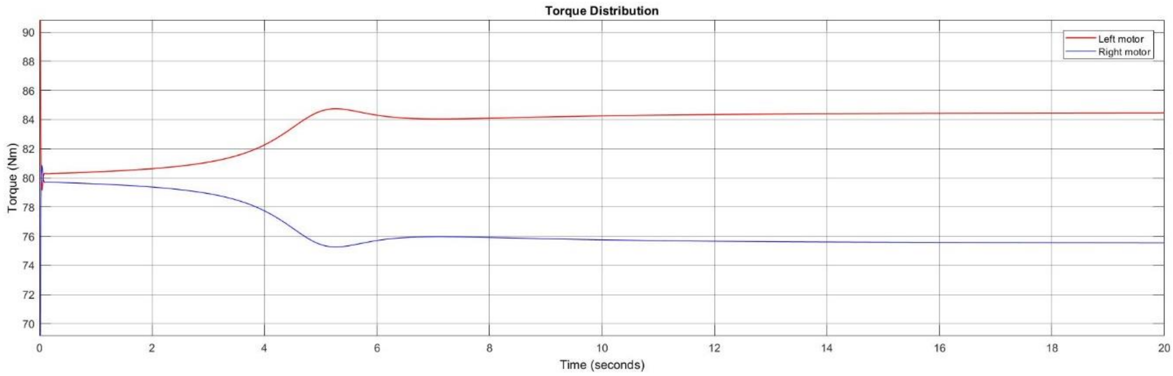

Figure 31. Torque response to J-turn input

The desired yaw rate is almost perfectly followed and no significant delays or relative errors can be seen in the yaw rate output, what means that the system is enough quick to process al data coming from the steering input and correct any peak. A starting saturation of torques can be explained due to initial non zero values of input, even though this isolated problem, a peak in torque is happening the the desired yaw rate response is more steep according to a quick clockwise movement.

5.4 Skid pad circuit

As part of the dynamic tests presented in section 2.1.1, the Skid Pad test measures the maximum lateral g's the vehicle can withstand. Keeping a constant radius along the circuit, the driver is encouraged to be aggressive enough to know the vehicle limit. In the most unfavourable case, the centripetal force will beat in magnitude to the force that can generate the tires where they would work out of the friction circle. Empirically, the team has observed in previous seasons this phenomenon as the vehicle "leaves the trace" or more technical terminology, the vehicle has understeer.

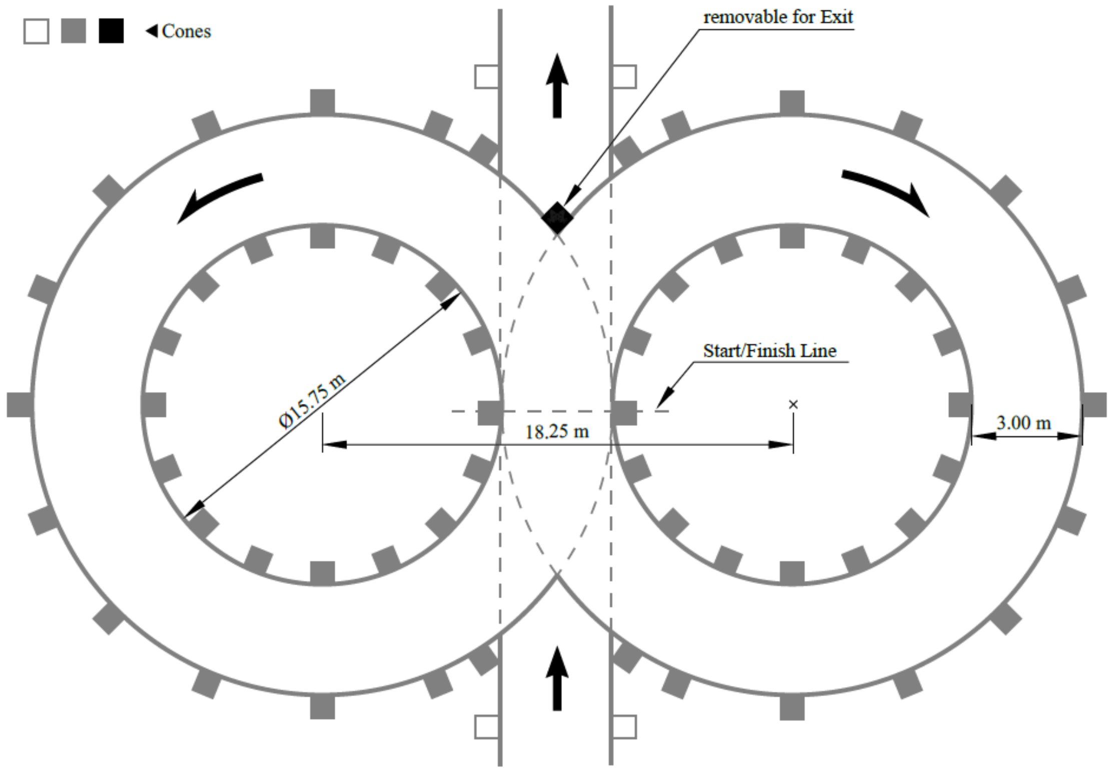

*Figure 32. SkidPad Circuit. **Source:** FS Germany Rules Handbook*

Coming from real data sensor from steering wheel, the steering input in computed. That signal represents a skid pad test the team did last season, therefore more realistic scenario cannot be done. In figure 33 and 34, the output responses of yaw rate and torque output is shown. Where the first very similar to the previous, the actual yaw rate follows to perfection the desired, no significant delays. Regarding the output torques, is clearly visible how the algorithm works with a 2 changes of direction. Between 10 to 28 seconds approximately the car is turning to left turn, so a adifferential torque difference between motors (16.26 Nm difference in wheels) is what the vehicle need to steer around Z axis. As no delay is shown in the output, it means the driver will expect what is currently happening in the car a good performance can be done.

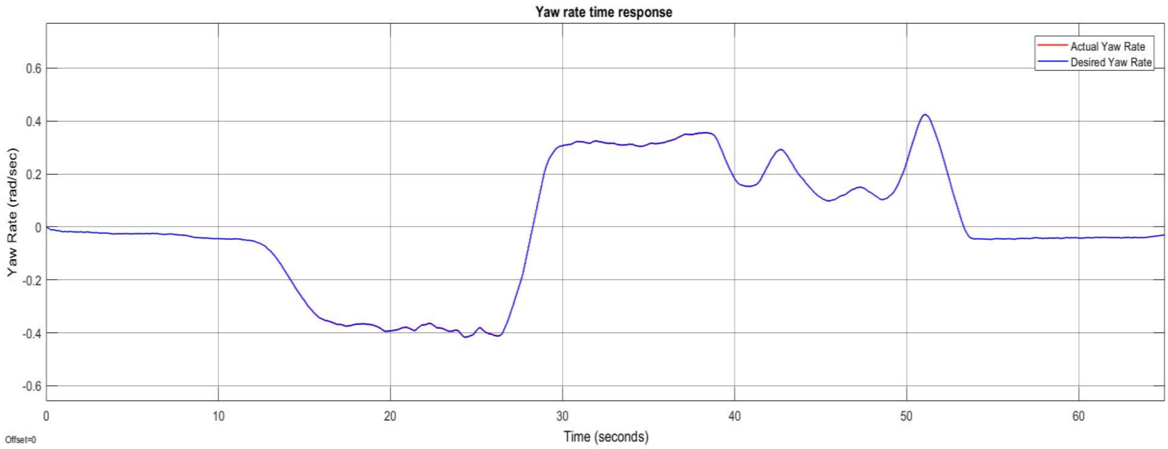

Figure 33. Yaw rate response to real steering input

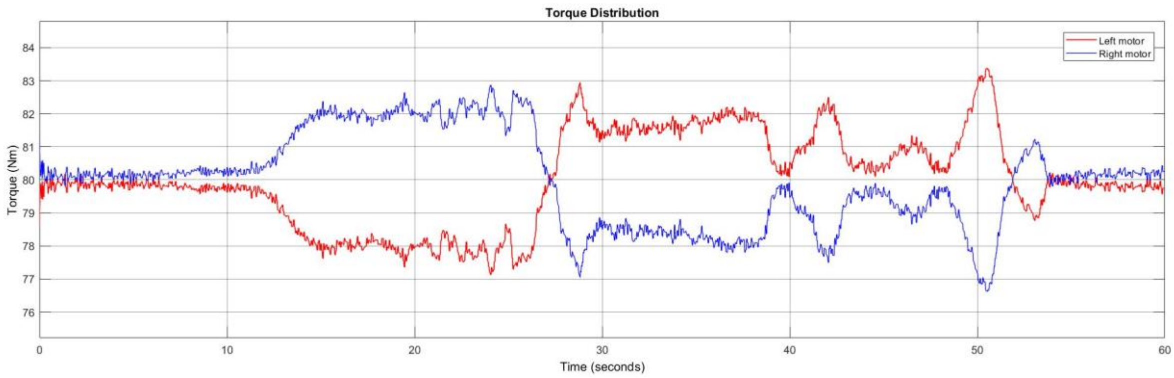

Figure 34. Torque response to real steering input

6. Conclusions and future work

The one part of goals of the thesis has been accomplished with this last chapter as a prove. A yaw rate control has been designed and it has been validated with hypothetical and real data. With the last scenario, where the real source of the sensor is the human, provides a noisy signal the algorithm has been able to control and not become instable.

As a drawback, the lack of a real simulator and assumptions done in the model, makes the output signal as a theory far from reality.

In order to make the System more realistic, the vehicle plant should be substitute by the vehicle model of any specific car Simulator, for example Car Maker from IPG Automotive. This step considers much more aspects that the equation of motion does, is possible to simulate any input signal and is fully suitable and customizable with Simulink too. This will result in a more accurate algorithm.

In case of real application, the current ECU of the team provided by ETAS is the model ES910.3 with real time execution could be used. Compiling in C code the Simulink file, it is very easy to import it to INTECRIO software (ES910.3 interface) and write it within the ECU. While testing, the variables of interest could be analysed and processed configuring the CAN bus net and incorporating a flash memory that could store all this data. And finally, tune the parameters according the feelings of the driver, the last step of the ladder of abstraction and vehicle modelling.

7. References

[1] Ghezzi, M. K. *Control of a four In-wheel motor drive electric vehicle.* Barcelona: Escola Tècnica Superior d'Enginyeria Industrial de Barcelona, Universitat Politènica de Barcelona. September, 2017. Industrial Engineering master thesis.

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[6] Lin, Cheng; Xu, Z. *Wheel torque distribution of four-wheel-drive electric vehicles based on multi-objective optimization.* Beijing, China: Collaborative Innovation Center of Electric Vehicles in Beijing, Beijing Institute of Technology. Energies 2015, 8, 3815-3831.

[7] Milliken, W. F; Milliken, D. L. *Race Car Vehicle Dynamics.* Warrandale, Pennsylvania, USA. Society of Automotive Engineers, 1995.

[8] Stoop, A. *Design and implementation of torque vectoring for the Forze Racing car: In collaboration with the Forze Hydrogen Racing team.* Delft: Mechanical, Maritime and Materials Engineering. Delft University of Technology. July 2, 2014. Master of Science Thesis.

[9] Zachery Brandstater. *Traction control and torque vectoring with Wheel hub motors.* School of Electrical, Electronic and Computer Engineering, University of Western Australia, November 2015. Bachelor thesis.

8. Budget

This thesis is a theoretical approach to the implementation of a dynamic control algorithm on the vehicle previously presented, an estimated budget of this project would cost is done. Simulations has been done with "free" software license provided by Mathworks, no cost is included. Only human resources hours of research, design and simulation are specified.

RECURSOS HUMANS

Description	Unitary cost	Quantity	Total cost
Concept and definition	10	20€/ h	200,00 €
Study and understanding references	300	20€/ h	6000,00 €
FSAE Germany rules reading	10	20€/ h	200,00 €
Car technical data	20	20€/ h	400,00 €
Design and simulations	200	20€/ h	4000,00 €
Thesis writing	250	20€/ h	5000,00 €
TOTAL			15800,00 €

Table 8. Cost estimation

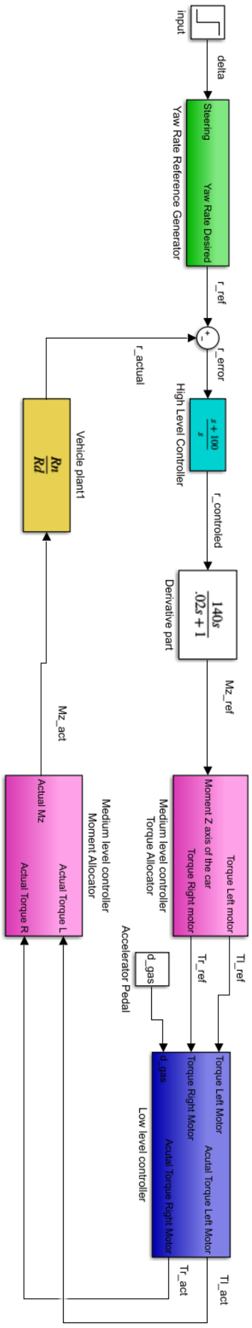
9. Annex

In this final Chapter is Presented the actual .m file and Simulink code which has been possible to do all simulations. The .m file is used only to initialize the variables for every scenario and charge them in Matlab Workspace, subsequently, Simulink file can be executed.

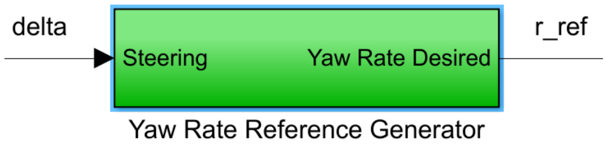
Matlat .m File

```
close all
clear all

%Declaration of variables
Cr = -1200;        %Cornering Stiffness rear track
Cf = -1000;        %Cornering Stiffness front track
m = 355;           %mass of the vehicle plus 80 kg of driver
V = 11;            %velocity vector of the CG [m/s]
b = 0.6;         %distance from rear track referred CG [m]
a = 0.7;         %distance from front track referred CG [m]
Iz = 140;          %inertia of the vehicle in the vertical axis
                    [kg/m*s^2]
d_gas = 1;         %Percentage accelerator pedal pressed
reff = 0.23;       %Effective radius of the tire
r_trans = 4.065; %Transmission Ratio

%Ecuation of motion derivatives
n_beta = a*Cf - b*Cr;
n_r = (1/V)*(a^2*Cf+b^2*Cr);
n_delta = -a*Cf;

y_beta = Cf+Cr;
y_r = (1/V)*(a*Cf+b*Cr);
y_delta = -Cf;

%Transfer function of the model
A = m*V;
B = y_beta;
C = m*V*Iz;
D = m*V*n_r+y_beta*Iz;
E = y_beta*n_r-n_beta*y_r+m*V*n_beta;
R = tf ([A -B],[C -D E]);
Rn = [A -B];
Rd = [C -D E];
```

Simulink File

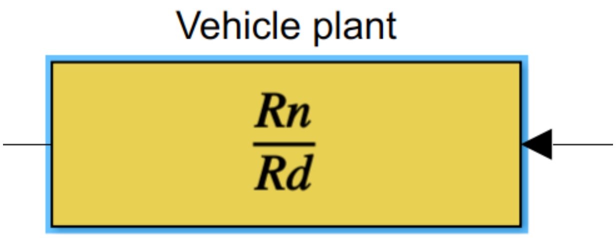


9.1.1. Yaw Reference Generator



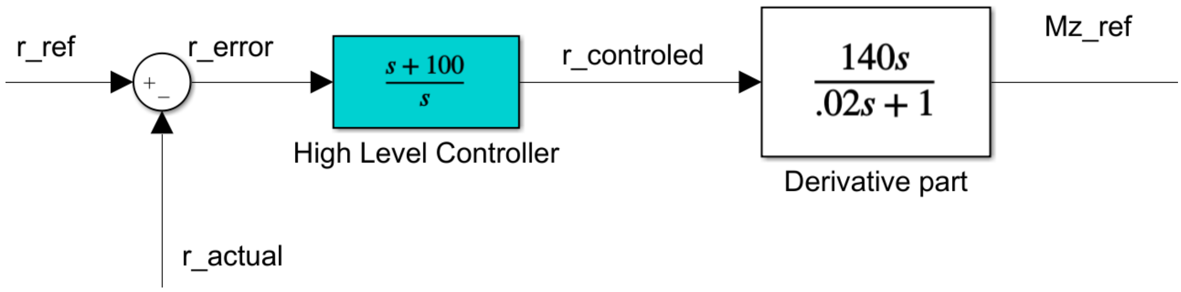

```
function yaw_des = fcn(steer)
l= a+b;
Ku = ((n_beta/-y_beta)/(n_delta-
(n_beta*y_delta/y_beta)))*m/trade;
yaw_des = -((V/trade)/(1+Ku*V^2))*steer;
```

9.1.2. Vehicle Plant



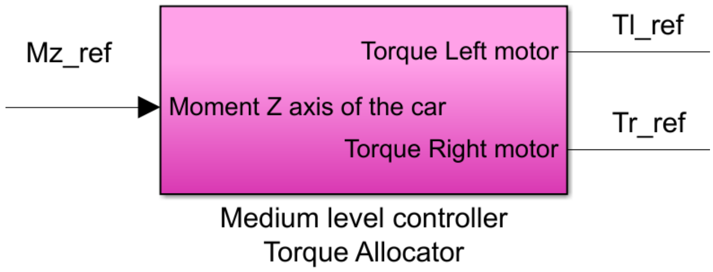

```
%In the .mat file, the values are initialized
Rn = [A -B];
Rd = [C -D E];
```

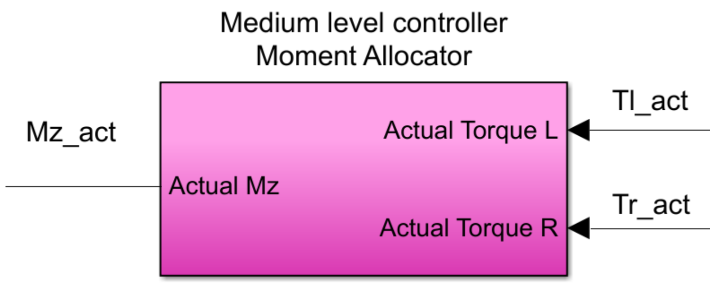
9.1.3. High Level Controller



Transfer Function Blocks of Simulink. Added a little constant of time to “Derivative Part Block” in order to Simulink allow simulation without singularities problems.

9.1.4. Medium Controller





Matlab function of Torque allocator:

```
function [Mm_l, Mm_r]= fcn(Mz)
%#codegen
Fx_l = 2*Mz/track;
Fx_r = -2*Mz/track;
Mw_l = Fx_l*reff;
Mw_r = Fx_r*reff;
```

```
Mm_l = Mw_l/r_trans;
Mm_r = Mw_r/r_trans;
```

lMatlab function or Moment Allocator:

```
function Mz = fcn(t_l,t_r)
%#codegen
Mw_l = t_l*r_trans  ;
Mw_r = t_r*r_trans  ;
Fx_l = Mw_l/reff  ;
Fx_r = Mw_r/reff  ;
Mz = Fx_l*track/2-Fx_r*track/2
```

9.1.5. Low Level Controller